Math 10550, Final makeup Exam, December 16, 2005

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on this page.
- Be sure that you have all 25 problems.
- This is the only page you need to hand in.

Please mark your answers with an X! Do NOT circle them!

<p>| | | | | | |</p>
<table>
<thead>
<tr>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
<th></th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>2</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>3</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>4</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>5</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>10</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>11</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>12</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>13</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>14</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>15</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>16</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>17</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>18</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>19</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>20</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>21</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>22</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>23</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>24</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>25</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

Final Exam: 

Previous Total: 

Course Total:
1. (6 pts.) Compute $\lim_{x \to 2} \frac{x^2 - 4}{x^2 - 5x + 6}$.

(a) $-4$  (b) $\infty$  (c) 0  (d) 1  (e) $-\infty$

2. (6 pts.) Compute $\lim_{x \to 0^+} \frac{x^2 - 9}{\sin x}$.

(a) Does not exist and is not $\infty$ or $-\infty$.  (b) $\infty$

(c) 0  (d) $-9$

(e) $-\infty$
3. (6 pts.) Evaluate \( \lim_{x \to \infty} (\sqrt{x^2 - x} - \sqrt{x^2 + 5x}) \).

(a) 0  \hspace{1cm} (b) 3  \hspace{1cm} (c) -6

(d) Does not exist  \hspace{1cm} (e) -3

4. (6 pts.) For what constant \( a \) is the function \( f \) given by

\[
f(x) = \begin{cases} 
ax + 1 & x < 0 \\
x^2 + 1 & x \geq 0 
\end{cases}
\]
differentiable everywhere?

(a) \( a = 2 \)  \hspace{1cm} (b) \( a = 0 \)

(c) \( a = 1 \)  \hspace{1cm} (d) Any value of \( a \)

(e) No value of \( a \)
5. (6 pts.) Compute \( \lim_{{x \to 0}} \frac{\tan 2x}{\sin 3x} \).

(a) 1/3  (b) 2  (c) 0  (d) 2/3  (e) 1

6. (6 pts.) Compute \( \lim_{{x \to \infty}} \frac{\sqrt{4x^2 + x + 1}}{3x - 1} \).

(a) 0  (b) -2/3  (c) 2/3  (d) 1/3  (e) -1/3
7. (6 pts.) Compute the tangent line to the ellipse given by the equation \( x^2 + 4y^2 = 5 \) at the point \((1, -1)\)

(a) \( y = \frac{1}{2}x - \frac{3}{2} \)

(b) The tangent line does not exist.

(c) \( y = \frac{1}{4}x - \frac{5}{4} \)

(d) \( y = \frac{1}{4}x - \frac{3}{4} \)

(e) \( y = -\frac{1}{4}x - \frac{3}{4} \)

8. (6 pts.) Let \( F(x) = f(g(x)) \). Compute \( F'(2) \) using the following information:

\( f(-1) = -3, f(2) = 12, g(-1) = -7, g(2) = -1, \)
\( f'(-1) = 2, f'(2) = 8, g'(-1) = -1, g'(2) = 5. \)

(a) 10  
(b) -15  
(c) 40  
(d) 2  
(e) 52
9. (6 pts.) For \( y = (\sin 4x)^8 \), compute \( y' \).

(a) \( 32(\cos 4x)^7 \)  
(b) \( 8(\cos 4x)^7 \)

(c) \( 8(\sin 4x)^7 \)  
(d) \( 32(\sin 4x)^7 \)

(e) \( 32(\sin 4x)^7 \cos 4x \)

10. (6 pts.) How many inflection points does the curve \( y = \frac{x^5}{5} + \frac{x^4}{4} \) have?

(a) 1  
(b) 0  
(c) 3  
(d) 2  
(e) 4
11. (6 pts.) Compute the derivative $y'$ for the curve $\sqrt{x^2 + y^2} = 2 + y$ at the point $x = 4$, $y = 3$.
(a) $\frac{2}{11}$  (b) $-2$  (c) $2$  (d) $0$  (e) $-\frac{2}{11}$

12. (6 pts.) A kite 100 ft above the ground is flying horizontally (away from its holder) with a speed of 16 ft/sec. At what rate is the angle between the string and the horizontal direction changing, when 200 ft of the string have been let out?
(a) $\frac{\pi}{50}$ radian/second  (b) $\frac{1}{25}$ radian/second
(c) $\frac{1}{50}$ radian/second  (d) $-\frac{1}{25}$ radian/second
(e) $-\frac{1}{50}$ radian/second
13. (6 pts.) Find the linearization of \( f(x) = \sqrt{10 - x^2} \) at \( a = -1 \).

(a) \( L(x) = \frac{2}{3} (x + 1) + 3 \)  
(b) \( L(x) = -\frac{2}{3} (x + 1) + 3 \)

(c) \( L(x) = x + 4 \)  
(d) \( L(x) = -\frac{1}{3} (x + 1) + 3 \)

(e) \( L(x) = \frac{1}{3} (x + 1) + 3 \)

14. (6 pts.) Find all local maxima and minima of the function \( f(x) = 2|x| - x^2 - 1 \).

(a) Local maxima: \((x, y) = (-1, 0)\) and \((x, y) = (1, 0)\), local minimum \((x, y) = (0, -1)\).

(b) Only local minimum at \((x, y) = (0, -1)\), no local maxima.

(c) Local maximum: \((x, y) = (-1, 0)\), local minimum \((x, y) = (0, -1)\).

(d) No local maxima or minima, because the function \(|x|\) has no derivative at \(x = 0\).

(e) Local maxima: \((x, y) = (-1, 0)\) and \((x, y) = (1, 0)\), no local minimum.
15. (6 pts.) Find all asymptotes of the curve \( y = \frac{2x^2 + x + 1}{x - 1} \).

(a) vertical asymptote \( x = 1 \), no other asymptotes.

(b) slant asymptote \( y = 2x + 1 \), vertical asymptote \( x = 1 \), no horizontal asymptotes.

(c) horizontal asymptotes \( y = 2 \), slant asymptote \( y = 2x + 3 \), no vertical asymptotes.

(d) slant asymptote \( y = 2x + 3 \), vertical asymptote \( x = 1 \), no horizontal asymptotes.

(e) horizontal asymptotes \( y = 2 \), vertical asymptote \( x = 1 \), no slant asymptotes.

16. (6 pts.) Find all the points on the hyperbola \( y^2 - x^2 = 4 \) that are closest to the point \((2, 0)\).

(a) \((1, \pm 5)\)  
(b) \((1, \pm \sqrt{5})\)  
(c) \((-1, \sqrt{5})\)

(d) \((1, \sqrt{5})\)  
(e) \((\sqrt{5}, 1)\)
17. (6 pts.) A page of a book is to have a total area of 150 square inches, with 1 inch margins at the top and sides, and a 2 inch margin at the bottom. Find the dimensions in inches of the page which will have the largest print area.

(a) \(3\sqrt{7} \times \frac{50}{\sqrt{7}}\)  \hspace{1cm} (b) \(5 \times 30\)  \hspace{1cm} (c) \(11\frac{7}{13} \times 13\)

(d) \(5\sqrt{3} \times \frac{30}{\sqrt{3}}\)  \hspace{1cm} (e) \(10 \times 15\)

18. (6 pts.) Newton’s method is to be used to find a root of the equation

\[x^3 - x - 1 = 0.\]

If \(x_1 = 1\), find \(x_2\).

(a) 1.50  \hspace{1cm} (b) 0.95  \hspace{1cm} (c) 3  \hspace{1cm} (d) 1.35  \hspace{1cm} (e) 1.75
19. (6 pts.) Express the limit below as a definite integral.

\[
\lim_{n \to \infty} \sum_{i=1}^{n} \frac{\pi}{4n} \sec^2 \left( \frac{i\pi}{4n} \right)
\]

(a) \( \int_{0}^{1} \sec^2 \left( \frac{\pi}{4} x \right) dx \) 
(b) \( \frac{\pi}{4} \int_{0}^{\pi/4} \sec^2(x) dx \)

(c) \( \int_{0}^{\pi/4} \sec^2 \left( \frac{\pi}{4} \right) dx \) 
(d) \( \int_{0}^{\pi/2} \sec^2(x) dx \)

(e) \( \int_{0}^{\pi/4} \sec^2(x) dx \)

20. (6 pts.) If \( f(x) = \int_{0}^{5x} \cos(u^2) du \), find \( f'(x) \).

(a) \( -\cos(5x^2) \) 
(b) \( 5 \cos(25x^2) \) 
(c) \( -25 \cos(5x^2) \)

(d) \( 5 \cos(5x^2) \) 
(e) \( -5 \cos(25x^2) \)
21. (6 pts.) Evaluate the integral \( \int_0^{\sqrt{\pi}} x \sin(x^2) \, dx \).

(a) \( \frac{\pi}{4} \)  \hspace{1cm} (b) 2  \hspace{1cm} (c) \( \frac{1}{4} \)  \hspace{1cm} (d) 1 \( - \frac{1}{\pi} \)  \hspace{1cm} (e) 1

22. (6 pts.) Which of the following integrals give the area of the region below the curve \( y = 2x \) and above the curve \( y = x^2 - 4x \)?

(a) \( \int_0^4 \left( (x^2 - 4x) - 2x \right) \, dx \)

(b) \( \int_0^6 \left( (x^2 - 4x) - 2x \right) \, dx \)

(c) \( \int_0^6 \left( 2x - (x^2 - 4x) \right) \, dx \)

(d) \( \int_0^4 \left( 2x - (x^2 - 4x) \right) \, dx \)

(e) \( \int_0^4 \left( 2x - (x^2 - 4x) \right) \, dx + \int_4^6 \left( (x^2 - 4x) - 2x \right) \, dx \)
23. (6 pts.) An area in $xy$ plane bounded by the curves $y = 0$ and $y = x - x^2$. If we rotate this area about $x = 7$, which integral below gives the volume?

(a) $\pi \int_0^{1/4} (x - x^2)^2 \, dx$
(b) $2\pi \int_0^1 (7-x)(x-x^2) \, dx$
(c) $2\pi \int_0^1 (x - x^2 - 7) \, dx$
(d) $\pi \int_0^1 (x-x^2)^2 \, dx$
(e) $2\pi \int_0^1 (x-7)(x-x^2) \, dx$

24. (6 pts.) The plane region bounded by the curves $y = 2$ and $y = 2 + 2x - x^2$ is rotated about the $x$ axis. Which integral below gives the volume?

(a) $\pi \int_0^2 \left(4 - (2 + 2x - x^2)^2\right) \, dx$
(b) $\pi \int_0^2 \left((2+2x-x^2)^2 - 4\right) \, dx$
(c) $2\pi \int_0^2 \left((2+2x-x^2) - 2\right) \, dx$
(d) $\pi \int_0^1 \left((2+2x-x^2)^2 - 4\right) \, dx$
(e) $\pi \int_0^1 \left(4 - (2 + 2x - x^2)^2\right) \, dx$
25. (6 pts.) The function $f(x) = \sqrt{16 - 2x}$ is continuous on the interval $[0, 8]$. Which number below is its average value on this interval?

(a) $\frac{8}{3}$  
(b) $\frac{64}{3}$  
(c) $\frac{8}{3}\sqrt{8}$

(d) $\frac{16}{3}$  
(e) $-\frac{8}{3}$
Math 10550, Final makeup Exam, December 16, 2005

- The Honor Code is in effect for this examination. All work is to be your own.
- No calculators.
- The exam lasts for 2 hours.
- Be sure that your name is on this page.
- Be sure that you have all 25 problems.
- This is the only page you need to hand in.

Please mark your answers with an X! Do NOT circle them!