Record your answers to the multiple choice problems by placing an × through one letter for each problem on this answer sheet. There are 20 multiple choice questions. Please sign the honor statement if you agree:

“I strictly followed the Notre Dame Honor Code during this test.”

Your Signature ____________________
1. Find the maximum rate of change of \( f(x, y) = x^2y + 2y \) at the point \((-1, 2)\) and the direction in which it occurs.

(a) The maximum rate of change is 5 in the direction \( \frac{1}{5}(-4, 3) \).

(b) The maximum rate of change is 5 in the direction \( \frac{1}{5}(4, -3) \).

(c) The maximum rate of change is 5 in the direction \( \frac{1}{5}(4, 3) \).

(d) The maximum rate of change is 6 in the direction \( \frac{1}{5}(-4, 3) \).

(e) The maximum rate of change is 5 in the direction \( \frac{1}{5}(-3, 4) \).
2. Let \( S \) be the part of cylinder \( y^2 + z^2 = 1 \), with \( z \geq 0 \), and \( 0 \leq x \leq 1 \), and let \( S \) have the upward orientation. Determine which of the following equals \( \int \int_S \mathbf{F} \cdot dS \) where \( \mathbf{F}(x, y, z) = (0, 0, z) \).

(a) \( \int_0^1 \int_{-1}^1 \sqrt{1 - y^2} \, dy \, dx \)  
(b) \( \int_0^1 \int_{-1}^1 (1 - y^2) \, dy \, dx \)

(c) \( \int_0^1 \int_{-1}^1 (1 - x^2) \, dy \, dx \)  
(d) \( \int_0^1 \int_{-1}^1 [\sqrt{1 - y^2}]^{-1} \, dy \, dx \)

(e) \( \int_0^1 \int_{-1}^1 \sqrt{1 - x^2} \, dy \, dx \)
3. Let $C$ be the triangle with vertices $(0, 0)$, $(1, 1)$, and $(2, 0)$ oriented counterclockwise. Compute

$$\int_C [\cos x^{100} + x^4 y^5] \, dx + [\sin(e^y) + x^5 y^4] \, dy$$

(a) 0  (b) $\frac{5}{4}$  (c) $-1$  (d) $\frac{4}{5}$  (e) 1

4. Let $C$ be the curve $\mathbf{r}(t) = (t, \cos(2t), 1 + \sin(3t))$, $0 \leq t \leq \frac{\pi}{2}$, and let

$$\mathbf{F}(x, y, z) = (y(2x + z), x(x + z) - z, y(x - 1) + 2z).$$

Compute $\int_C \mathbf{F} \cdot d\mathbf{r}$. (Hint: Find $f$ with $\mathbf{F} = \nabla f$).

(a) $-\frac{\pi^2}{4}$  (b) $\frac{\pi^2}{2} - 1$  (c) $\frac{\pi}{2}$  (d) 0  (e) $1 - \frac{\pi}{4}$
5. Find the absolute minimum of \( f(x, y) = x^2 + 2y^2 + 4y - 2 \) on the disk \( x^2 + y^2 \leq 4 \).

(a) -4  (b) -6  (c) 0  (d) -2  (e) 14

6. Find the scalar projection, \( \text{comp}_v(w) \), of the vector \( w = (1, 1, 2) \) onto the vector \( v = (2, -2, 1) \).

(a) 2  (b) \( \frac{2}{3} \)  (c) \( \frac{2}{\sqrt{6}} \)  (d) 1  (e) \( -\frac{2}{3} \)
7. Determine two vectors that are tangent to the surface \( \mathbf{r}(u, v) = (vu^2 - 2u, uv^2 - v, uv) \) at the point \((0, 1, 2)\).

(a) \((0, 1, 1), (4, 1, 1)\)  
(b) \((2, 4, 2), (1, 3, 1)\)  
(c) \((2, 1, 1), (4, 3, 2)\)

(d) \((0, 2, -1), (1, -6, 3)\)  
(e) \((1, 2, -1), (1, -2, 1)\)

8. Evaluate \(\iiint_{E} 3e^{[(x^2 + y^2 + z^2)^{3/2}]} dV\) where \(E\) is the upper half of the ball radius 1 centered at the origin.

(a) \(\pi\)  
(b) \(4\pi(e - 1)\)  
(c) \(2\pi\)  
(d) \(2\pi(e - 1)\)  
(e) \(\pi(e - 1)\)
9. If \( z = f(x, y) \), \( x = u^2 + v^2 \) and \( y = u^2 - v^2 \), find \( \frac{\partial^2 z}{\partial u \partial v} \).

(a) \( 4uv(f_{xx} - f_{yy}) \)  
(b) \( 4uv(f_{xx} + 2f_{yy} + f_{yy}) \)  
(c) \( 2(f_{xx} + f_{yy}) \)

(d) \( 4uv(f_{xx} + f_{yy}) \)  
(e) \( 4uv(f_{xx} + 2f_{yy} - f_{yy}) \)

10. Find a direction vector for the line of intersection of the planes \( x + y + 2z = 1 \) and \( 3x - y = 0 \).

(a) \( \langle 1, 3, -2 \rangle \)  
(b) \( \langle 3, 1, -2 \rangle \)  
(c) \( \langle 3, -2, 1 \rangle \)  
(d) \( \langle -2, 3, 1 \rangle \)  
(e) \( \langle 1, 3, 2 \rangle \)
11. Calculate the arc length of the helix parameterized by \( \mathbf{r} = (-3t, 4 \cos t, -4 \sin t) \) for \( 0 \leq t \leq \pi \\
(a) 12\pi \quad (b) 2\pi \quad (c) 5\pi \quad (d) 0 \quad (e) 10\pi \\

12. Find \( \int \int_S \mathbf{F} \cdot d\mathbf{S} \) where \( \mathbf{F}(x, y, z) = (xy, \frac{3}{4}y, -zy) \) and \( S \) is the surface of the sphere \( x^2 + y^2 + z^2 = 4 \) with the outward orientation.

(a) 2\pi \quad (b) 16\pi \quad (c) \pi \quad (d) 0 \quad (e) 8\pi
13. Express the area between \( x^2 + \frac{y^2}{9} = 1 \) and \( x^2 + \frac{y^2}{9} = 9 \) as an integral, using the substitution \( x = r \cos \theta \) and \( y = 3r \sin \theta \).

(a) \( \int_0^{2\pi} \int_0^3 3rdrd\theta \)  
(b) \( \int_0^{2\pi} \int_1^3 9rdrd\theta \)  
(c) \( \int_0^{2\pi} \int_1^3 3rdrd\theta \)  
(d) \( \int_0^{2\pi} \int_1^3 3r^2drd\theta \)  
(e) \( \int_0^{2\pi} \int_1^9 3rdrd\theta \)

14. Let \( C \) be the intersection of the cylinder \( x^2 + y^2 = 1 \) and the plane \( z = 2y + 3 \) oriented counter-clockwise with the normal upwards. Use Stokes Theorem to calculate \( \int_C \mathbf{F} \cdot d\mathbf{r} \) where

\[
\mathbf{F}(x, y, z) = (2e^y - z, \cos(yz), xe^y).
\]

(a) 0  
(b) \( \frac{\pi}{\sqrt{5}} \)  
(c) \( 2\pi \)  
(d) \( \sqrt{5}\pi \)  
(e) \( \frac{2\pi}{\sqrt{5}} \)
15. Determine which of the following integrals gives the area of the region in the $xy$-plane below the $x$-axis above $y = x^2 - 2$ and to the left of $y = -2x - 2$.

(a) $\int_{-\sqrt{2}}^{0} \int_{-\sqrt{y+2}}^{\frac{-1}{2}} dx
ddy$(b) $\int_{-2}^{0} \int_{\sqrt{y+2}}^{\frac{-1}{2}} dx
ddy$(c) $\int_{-2}^{0} \int_{-2x-2}^{x^2-2} dy
ddx$(d) $\int_{-2}^{0} \int_{-\sqrt{y+2}}^{\frac{1}{2}} dx
ddy$(e) $\int_{-2}^{0} \int_{\sqrt{y+2}}^{\frac{-1}{2}} dx
ddy

16. Evaluate $\int_{C} (1 + x^2y)ds$ where $C$ is the upper half of the unit circle $x^2 + y^2 = 1$.

(a) 1.  (b) $\frac{2}{3}$.
(c) 0.
(d) $2\pi$.
(e) $\pi + \frac{2}{3}$.
17. Determine which of the following integrals gives the volume of the region bounded by the cylinder \( x^2 + y^2 = 1 \), and the planes \( z = 0 \) and \( x + z = 1 \).

(a) \( \int_0^{2\pi} \int_0^1 \int_0^{1-r \cos \theta} rdzdrd\theta \)

(b) \( \int_0^1 \int_0^{\sqrt{1-y^2}} \int_0^{1-x} dzdxdy \)

(c) \( \int_0^{2\pi} \int_0^1 \int_0^{1-r \cos \theta} dzdrd\theta \)

(d) \( \int_0^{\pi/2} \int_0^1 \int_0^{1-r \cos \theta} rdzdrd\theta \)

(e) \( \int_0^\pi \int_0^1 \int_0^{1-r \cos \theta} rdzdrd\theta \)
18. Find surface area of the part of the paraboloid \( z = x^2 + y^2 \) that lies under the plane \( z = 4 \).

(a) \( \int_0^{2\pi} \int_0^2 r \sqrt{1 + 4r^2} \, dr \, d\theta \)  
(b) \( \int_0^{2\pi} \int_0^1 r \sqrt{1 + 4r^2} \, dr \, d\theta \)

(c) \( \int_0^{2\pi} \int_0^1 r \sqrt{1 + 4r^2} \, dr \, d\theta \)  
(d) \( \int_0^{\pi} \int_0^2 r \sqrt{1 + 4r^2} \, dr \, d\theta \)

(e) \( \int_0^{2\pi} \int_0^2 \sqrt{1 + 4r^2} \, dr \, d\theta \)

19. A particle starts from rest at time \( t = 0 \) at the origin \((0,0,0)\). It then begins to move with acceleration \( \mathbf{a}(t) = (1, 6t, 12t^2) \). Find the time, if ever, at which the particle passes through the point \((1,1,1)\).

(a) never  
(b) \( t = 2 \)  
(c) \( t = 6 \)  
(d) \( t = 3 \)  
(e) \( t = 1 \)
20. Find the minimum of the function \( f(x, y, z) = x^2 + y^2 + 2z^2 \) on the surface \( x^2y^2z = 16 \).

(a) 14  (b) 12  (c) 8  (d) 16  (e) 10
Record your answers to the multiple choice problems by placing an \( \times \) through one letter for each problem on this answer sheet. There are 20 multiple choice questions. Please sign the honor statement if you agree:

“I strictly followed the Notre Dame Honor Code during this test.”

Your Signature ____________________________

1. \( \bullet \quad b \quad c \quad d \quad e \)  
2. \( \bullet \quad b \quad c \quad d \quad e \)  
3. \( \bullet \quad b \quad c \quad d \quad e \)  
4. \( \bullet \quad b \quad c \quad d \quad e \)  
5. \( \bullet \quad b \quad c \quad d \quad e \)  
6. \( a \quad \bullet \quad c \quad d \quad e \)  
7. \( a \quad b \quad \bullet \quad d \quad e \)  
8. \( a \quad b \quad c \quad \bullet \quad e \)  
9. \( \bullet \quad b \quad c \quad d \quad e \)  
10. \( \bullet \quad b \quad c \quad d \quad e \)  
11. \( a \quad b \quad \bullet \quad d \quad e \)  
12. \( a \quad b \quad c \quad d \quad \bullet \)  
13. \( a \quad b \quad \bullet \quad d \quad e \)  
14. \( a \quad b \quad \bullet \quad d \quad e \)  
15. \( a \quad b \quad c \quad d \quad \bullet \)  
16. \( a \quad b \quad c \quad d \quad \bullet \)  
17. \( \bullet \quad b \quad c \quad d \quad e \)  
18. \( \bullet \quad b \quad c \quad d \quad e \)  
19. \( \bullet \quad b \quad c \quad d \quad e \)  
20. \( a \quad b \quad c \quad d \quad \bullet \)